Effect of a Volume Heat Source on Free-Convection Heat Transfer

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Heat transfer to a horizontal cylinder from an ordinary fluid containing a volume heat source was studied analytically. The first approach to this problem was to assume that the heat source existed only in the boundary layer surrounding the cylinder. By solving the equations for a vertical flat plate cooling the ambient fluid and applying the results to a horizontal cylinder, one could determine the effect of a volume heat source on the heat transfer coefficient for the cylinder case. The variables studied were volume heat sources from 0 to 300 B.t.u./(sec.)(cu. ft.) and temperature difference across the boundary layer in the range 20° to 100°F.

In the laminar-flow region a volume heat source was found to increase the heat transfer coefficient for any given temperature difference across the boundary layer. This increase, due to the decrease in boundary-layer thickness caused by the volume heat source, was moderate, being about 20% for a $1\frac{1}{4}$-in. O.D. cylinder with a heat source strength of 100 B.t.u./(sec.)(cu. ft.) and a temperature difference across the boundary layer of 20°F.

PROBLEM ANALYSIS

The model considered was a horizontal cylinder immersed in an infinite fluid containing a volume heat source. The cylinder surface temperature was maintained lower than the temperature of the surrounding fluid by having a coolant passed through it, and the volume-generated heat thus removed. Since there was no impressed flow on the bulk fluid body, this cooling took place by free convection.

Free-convection heat transfer problems are generally solved with the simultaneous use of the continuity equation, an energy balance, and the appropriate momentum balances in conjunction with boundary-layer theory. The inclusion of a volume heat source in such a problem, however, complicates the physical picture, since at steady state conditions this heat generated at all points in the fluid will have to be removed, necessitating a temperature gradient. Since the fluid density is a function of temperature, a velocity gradient will also be generated throughout the fluid. Boundary-layer theory though assumes that all the temperature and velocity gradients occur in a layer near the body surface, with the rest of the body isothermal and at constant velocity. Since this is not the case with a volume heat source present, the boundary-layer assumptions are not strictly applicable, and their use constitutes an approximation to the true picture.

Many attempts were made therefore to find a suitable model, one in which the boundary-layer theory need not be used. The initial step was to investigate boundary conditions for the solution. Utilizing the no slip at the tube wall assumption, the authors established the velocity at the wall as zero. Two thermal-boundary conditions were possible however. Either the heat flux at the wall or the temperature variation of the wall could be specified but not both. For the problem under consideration a constant wall temperature was specified with the remainder of the boundary condition necessary for solution dependent on the model used.

At first the cylinder was assumed to be immersed in an infinite domain, a fluid which extends to infinity in every direction. With a volume heat source present, however, the velocity and temperature profiles theoretically do not become constant and hence would have to be known to infinity. This model was therefore rejected because of boundary-condition difficulties.

A model which appeared promising was one having a concentric, perfectly insulated outer cylinder surrounding the tube. Although with this model zero velocity and zero temperature gradient ($\partial T/\partial r$) could be specified at the outer wall, a solution to the equations could not be obtained because of mathematical difficulties encountered; so this model was dropped.

Concurrent with this analytical study experimental work was being carried out as part of the over-all project. While attempts were being made to circumvent the boundary-layer assumptions, tests showed that the temperature and velocity gradients which were expected through the fluid were negligible relative to the larger gradients in the immediate vicinity of the tube (*1*). It therefore appeared that the boundary-layer assumptions were valid, and the boundary-layer approximations could be used.

On this evidence a boundary-layer solution, with the assumption that the heat source was present only in the boundary layer, was attempted. Since Hermann (*4*) presented an exact analytical solution for free convection heat transfer to air having no volume heat source, a check was available for the iteration technique attempted for solution with an Electrodata digital computer. Hermann's equations were converted to difference form and his results used as initial conditions for the iterations. Thus if the iteration technique were to work, the iterations should have converged immediately to the initial conditions. Once this proof of the technique was completed, Hermann's equations could be altered to include the effect of the volume heat source, and the problem could be solved. The iterations however diverged, indicating the need for a lengthy step-size study. The following approximate solution was therefore attempted.

THEORETICAL SOLUTION DEVELOPMENT

A common technique in the solution of fluid flow-heat transfer problems involving cylinders is to solve a similar problem for a flat plate and by conformal mapping or some other procedure transform the results to the cylinder. This technique is valid owing to the boundary-layer character of the differential equations underlying the theoretical solutions for both cases (*4*). The technique is used because boundary-layer problems for the

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flat-plate geometry are more readily solvable than the ones for the outside of a cylinder. Thus to find heat transfer coefficients to the cylinder, both with and without heat source, flat-plate solutions could be obtained and then transformed to the cylinder. If however the ratio of the heat transfer coefficients for the flat plate were determined instead, this ratio could be applied directly to the cylinder, if the assumption is made that the transformation will introduce errors of equal relative magnitude to both coefficients. This was the procedure followed for this solution, which is described in detail by Randall (*10*).

The model used was a vertical plate cooling a surrounding infinite still fluid at a constant temperature everywhere except in the boundary layer. The assumptions made were

1. There are no effects longitudinally along the plate; that is the problem is two-dimensional.
2. There is no velocity component perpendicular to the surface of the plate.
3. The buoyant force is the only driving force for fluid flow.
4. The fluid will not slip at the wall.
5. The fluid velocity is zero at the outer edge of the boundary layer.
6. The surface temperature of the plate is constant all over the plate.
7. The thermal and momentum boundary layers are of equal thickness. This assumption is valid for fluids having a Prandtl number close to 1.0, a condition approached by water in the 300° to 600°F. range.

In addition Eckert's (*2*) temperature and velocity profiles were assumed in the boundary layer. These profiles are shown in Figure 1. The defining equations for them are

$$\phi = \theta\left(1 - \frac{y}{\delta}\right)^2$$

$$u = \omega \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2$$

The boundary conditions for this problem were

$$\text{at } y = 0, \quad u = 0, \quad \phi = \theta$$

$$\text{at } y = \delta, \quad u = 0, \quad \phi = 0$$

The assumed profiles were shown to be in good agreement with the experimental profiles of Schmidt and Beckmann (*7*).

Two balances were used to obtain the solution, a momentum balance and an energy balance. The momentum balance was obtained by the use of the von Karman approximate method shown in reference *2*.

$$\frac{d}{dx}\int_0^\delta u^2\,dy = g\beta\int_0^\delta \phi\,dy - \nu\left(\frac{du}{dy}\right)_{y=0} \qquad (1)$$

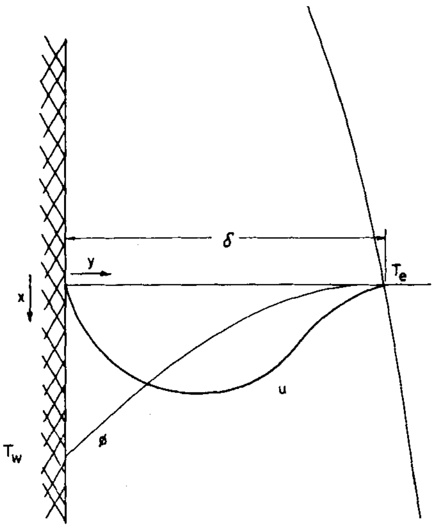

Fig. 1. Assumed velocity and temperature profiles.

In a similar fashion the energy balance was obtained:

$$\rho C_p \frac{d}{dx}\int_0^\delta T_e u\,dy - \rho C_p \frac{d}{dx}\int_0^\delta Tu\,dy + \int_0^\delta q'''\,dy + k\left(\frac{dT}{dy}\right)_{y=0} = 0$$

Dividing through by ρC_p one obtains

$$\frac{d}{dx}\int_0^\delta \phi u\,dy = -\alpha\left(\frac{d\phi}{dy}\right)_{y=0} - \frac{q'''\,\delta}{\rho C_p} \qquad (2)$$

since $\phi = Te - T$ and $\alpha = k/\rho C_p$

The volume heat-source term q''' can be taken out from under the integral sign, since it is considered uniform throughout the boundary layer and is therefore a constant.

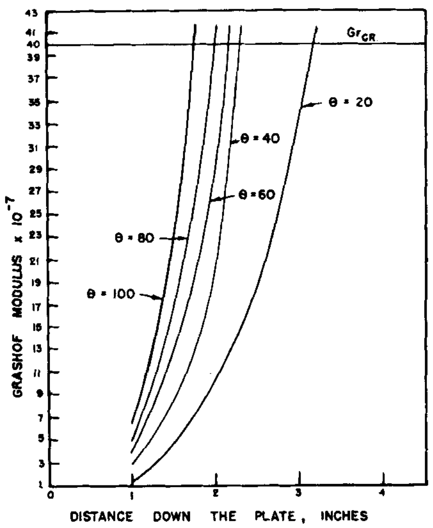

Fig. 2. Variation of Grashof modulus with *x* at parametric temperature differences.

The expressions for ϕ and u were then substituted into Equations (1) and (2) and integrated and differentiated. The resulting equations were

$$\frac{1}{105}\frac{d}{dx}(\omega^2\delta) = \tfrac{1}{3}g\beta\theta\delta - \nu\frac{\omega}{\delta} \qquad (3)$$

$$\frac{1}{30}\,\theta\frac{d}{dx}(\omega\delta) - \frac{2\alpha\theta}{\delta} + \frac{1}{\rho C_p}q'''\,\delta = 0 \qquad (4)$$

Next the equations were rendered dimensionless by defining the quantities

$$\Delta = \left(\frac{g\beta\theta}{\nu^2}\right)^{1/3}\delta$$

$$\Omega = (g\beta\theta\nu)^{-1/3}\omega$$

$$X = \left(\frac{g\beta\theta}{\nu^2}\right)^{1/3}x = (N_{Gr})^{1/3} \qquad (5)$$

$$K = \left[\frac{\nu^{1/3}}{(g\beta)^{2/3}\theta^{5/3}}\right]\frac{q'''}{\rho C_p}$$

When one substitutes expressions (5) into Equations (3) and (4), the equations become

$$\frac{1}{105}\frac{d}{dX}(\Omega^2\Delta) = \tfrac{1}{3}\Delta - \frac{\Omega}{\Delta} \qquad (6)$$

$$\frac{1}{30}\frac{d}{dX}(\Omega\Delta) - \frac{2}{N_{Pr}}\frac{1}{\Delta} + K\Delta = 0 \qquad (7)$$

This pair of simultaneous equations was then solved by a variation of the method of Frobenius (*8*).

Ω and Δ were expanded in the Maclaurin series.

$$\Omega(X, N_{Pr}, K) = \sum_{n=0}^{\infty} K^n\Omega_n(X, N_{Pr})$$

$$\Delta(X, N_{Pr}, K) = \sum_{n=0}^{\infty} K^n\cdot\Delta_n(X, N_{Pr}) \qquad (8)$$

Substituting these series into Equations (6) and (7) and collecting terms of like coefficient, one gets

$$K^0\left[\frac{1}{105}\frac{d}{dX}(\Omega_0^2\Delta_0) - \frac{\Delta_0}{3} + \frac{\Omega_0}{\Delta_0}\right] + K^1\left[\frac{1}{105}\frac{d}{dx}(\Omega_0^2\Delta_1 + 2\Omega_0\Omega_1\Delta_0) - \tfrac{1}{3}\Delta_1\frac{\Omega_1}{\Omega_0} - \frac{\Omega_0\Delta_1}{\Delta_0^2}\right] + K^2\left[\begin{matrix}\text{many termed}\\ \text{expression}\end{matrix}\right] + K^3\left[\begin{matrix}\text{many termed}\\ \text{expression}\end{matrix}\right] + \cdots = 0 \qquad (9)$$

$$K^0\left[\frac{1}{30}\frac{d}{dx}(\Omega_0\,\Delta_0) - \frac{2}{N_{Pr}}\frac{1}{\Delta_0}\right]$$

$$+ K^1\left[\frac{1}{30}\frac{d}{dx}(\Omega_0\,\Delta_2 + \Omega_1\,\Delta_1 + \Omega_2\,\Delta_0) - \frac{2}{N_{Pr}}\left[\frac{\Delta_1^{\,2}}{\Delta_0^{\,3}} - \frac{\Delta_2}{\Delta_0^{\,2}}\right] + \Delta_1\right]$$

$$+K^2\left[\begin{matrix}\text{many termed}\\ \text{expression}\end{matrix}\right]$$

$$+K^3\left[\begin{matrix}\text{many termed}\\ \text{expression}\end{matrix}\right] + \cdots = 0 \qquad (10)$$

If an entirely general solution were to be obtained for Equations (9) and (10), the quantities in the brackets had to be equal to zero. Therefore if the terms having like coefficients in Equations (9) and (10) were taken together, they formed a set of two simultaneous equations from which the values of Ω_n and Δ_n were calculated. For example the terms whose coefficient is K^0 are

$$\left\{\begin{aligned}&\frac{1}{105}\frac{d}{dX}(\Omega_0^{\,2}\,\Delta_0) - \frac{\Delta_0}{3} + \frac{\Omega_0}{\Delta_0} = 0\\ &\frac{1}{30}\frac{d}{dX}(\Omega_0\,\Delta_0) - \frac{2}{N_{Pr}}\frac{1}{\Delta_0} = 0\end{aligned}\right. \qquad (11)$$

A solution of the form

$$\left.\begin{aligned}\Omega_0 &= C_1X^m\\ \Delta_0 &= C_2X^n\end{aligned}\right\} \qquad (12)$$

was assumed.

When one substituted expressions (12) into (11), the following equations resulted:

$$\frac{2m+n}{105}C_1^{\,2}C_2X^{2m+n-1} - \frac{C_2X^n}{3} + \frac{C_1}{C_2}X^{m-n} = 0 \qquad (13)$$

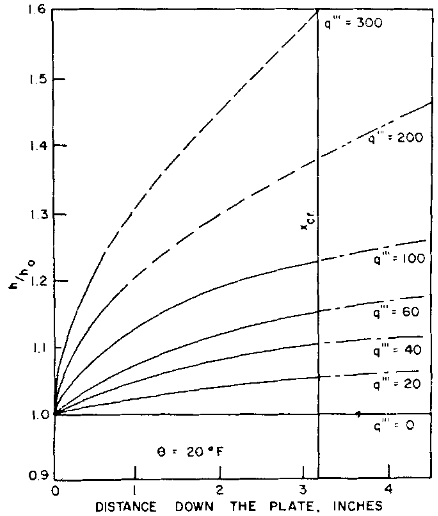

Fig. 3. Variation of h/h_0 with x for parametric volume heat sources.

$$\frac{m+n}{30}C_1C_2X^{m+n-1} - \frac{2}{C_2N_{Pr}}X^{-n} = 0 \qquad (14)$$

For a perfectly general solution of these equations the exponents of X for each equation must be equal.

Therefore

$$2m + n - 1 = n = m - n$$

$$m + n - 1 = -n$$

and m and n are

$$m = 1/2; \qquad n = 1/4$$

Substituting these values into Equations (13) and (14), one obtains

$$\left\{\begin{aligned}&\frac{1.25}{105}C_1^{\,2}C_2 - \frac{C_2}{3} + \frac{C_1}{C_2} = 0\\ &\frac{0.75}{30}C_1C_2 - \frac{1}{C_2}\frac{2}{N_{Pr}} = 0\end{aligned}\right.$$

The latter equations were solved simultaneously for C_1 and C_2 with the results

$$C_1 = 2[0.143 + 0.150N_{Pr}]^{-1/2}$$

$$C_2 = 3.94N_{Pr}^{-1/2}[0.953 + N_{Pr}]^{1/4}$$

Therefore

$$\Omega_0 = 2[0.143 + 0.150N_{Pr}]^{-1/2}X^{1/2}$$

$$\Delta_0 = 3.94N_{Pr}^{-1/2}[0.953 + N_{Pr}]^{1/4}X^{1/4}$$

A similar procedure was used to calculate Ω_1, Ω_2, Δ_1, and Δ_2. From Equations (9) and (10) it should be noted that to calculate values for Ω_n and Δ_n, the values of Ω_0 through Ω_{n-1} and Δ_0 through Δ_{n-1} must be known.

The results of these calculations were

$$\Omega_n = [g_n(N_{Pr})X^{n/2}]X^{1/2}$$

$$\Delta_n = [f_n(N_{Pr})X^{n/2}]X^{1/4}$$

and

$$\Omega = \sum_{n=0}^{\infty} K^n g_n(N_{Pr})X^{n/2}X^{1/2} \qquad (15)$$

$$\Delta_0 = \sum_{n=0}^{\infty} K^n f_n(N_{Pr})X^{n/2}X^{1/4} \qquad (16)$$

Values of g_n and f_n were calculated through $n = 2$, where the series were truncated.

It should be noted here that the solutions for Ω_0 and Δ_0 are quite similar to Eckert's (2) solutions for a vertical flat plate without volume heat source. The remainder of the terms in the respective series must express the effect of this source.

The following derivation was used to obtain the heat transfer coefficients.

Fourier's law states that

$$q = -k\left(\frac{d\phi}{dy}\right)_{y=0} \qquad (17)$$

Since ϕ is defined as

$$\phi = \theta\left(1 - \frac{y}{\delta}\right)^2$$

then

$$\left(\frac{d\phi}{dy}\right)_{y=0} = -\frac{2\theta}{\delta} \qquad (18)$$

Therefore

$$q = \frac{2k\theta}{\delta} \qquad (19)$$

The heat transfer coefficient is defined by the equation

$$q = h\theta$$

Substituting this in Equation (19) one got

$$h = \frac{2k}{\delta} \qquad (20)$$

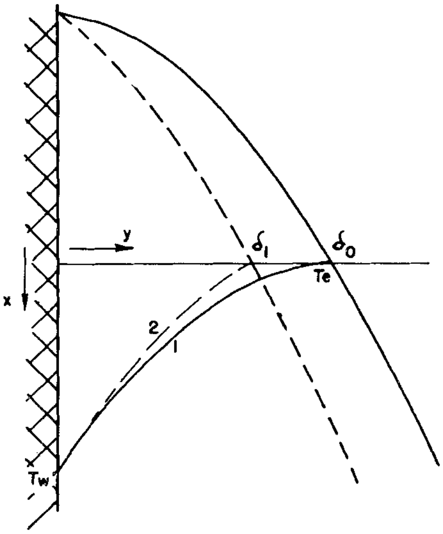

Fig. 4. Effect on the boundary-layer thickness of a volume heat source.

The ratio of the heat transfer coefficient with volume heat source to the coefficient without the heat source is therefore

$$\frac{h}{h_0} = \frac{\dfrac{2k}{\delta}}{\dfrac{2k}{\delta_0}} = \frac{\delta_0}{\delta}$$

Since Δ is defined as $= [(g\beta\theta)/\nu^2]^{1/3}\delta$, substituting in the corresponding Δ's one got

$$\frac{h}{h_0} = \frac{\Delta_0\left[\dfrac{g\beta\theta}{\nu^2}\right]^{-1/3}}{\Delta\left[\dfrac{g\beta\theta}{\nu^2}\right]^{-1/3}} = \frac{\Delta_0}{\Delta}$$

Therefore

$$\frac{h}{h_0} = \frac{C_2X^{1/4}}{\sum_{n=0}^{\infty} K^n f_n(N_{Pr})X^{n/2}X^{1/4}}$$

and

$$\frac{h}{h_0} = \frac{C_2}{\sum_{n=0}^{\infty} K^n f_n(N_{Pr}) X^{n/2}} \tag{21}$$

When one substitutes for C_2 the equivalent function of N_{Pr} and notes that $X = (N_{Gr})^{1/3}$, Equation (22) is obtained.

$$\frac{h}{h_0} = \frac{3.94 N_{Pr}^{-1/2}[0.953 + N_{Pr}]^{1/4}}{\sum_{n=0}^{\infty} K^n f_n(N_{Pr})[N_{Gr}]^{n/6}} \tag{22}$$

RESULTS

The effectiveness of the volume heat source could best be evaluated from the derived equations when one carries out a parametric study. Variables studied were

f_n are substituted into Equation (21), the equation becomes

$$\frac{h}{h_0} = \frac{3.88}{3.88 - 12.91KX^{1/2} + 46.5K^2X} \tag{23}$$

When one uses the physical properties, K and X become

$$K = \frac{\nu^{1/3} q'''}{(g\beta)^{2/3}\theta^{5/3}\rho C_p}$$

$$= 4.30 \times 10^{-3}\theta^{-5/3}q''' \tag{24}$$

$$X = \left[\frac{g\beta\theta}{\nu^2}\right]^{1/3} x = 1023\theta^{1/3}x$$

On substitution of expressions (24) into Equation (23), one obtained

$$\frac{h}{h_0} = \frac{3.88}{3.88 - 1.78q'''\theta^{-3/2}x^{1/2} + 0.886[q''']^2\theta^{-3}x} \tag{25}$$

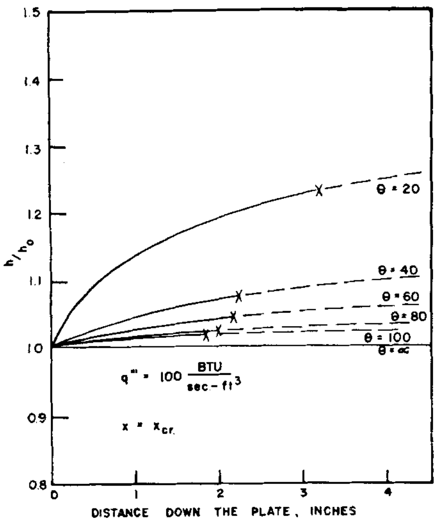

Fig. 5. Variation of h/h_0 with x for parametric temperature differences.

temperature difference, magnitude of the heat source, and tube size. The parameter study was conducted about a single fluid condition, 1 atm. and 200°F., where a number of properties are fairly constant as a function of temperature. Although a solution of some uranium compound would be necessary to provide the volume heat source, the properties of pure water at these conditions were assumed. The Prandtl number, which was not treated as a variable in this study, was equal to 1.70. With these data expressions (5) yielded

$$\Omega = 3.17X^{1/2} - 7.25KX - 6.17K^2X^{3/2}$$

$$\Delta = 3.88X^{1/4} - 12.91KX^{3/4} + 46.5K^2X^{5/4}$$

When the numerical values of C_2 and

When one used Equation (25), two sets of calculations were performed, one for θ constant and q''' as a parameter and the second for q''' constant and θ a parameter.

As a guide in the determination of applicable combinations of θ and x, where the flow in the boundary layer would be laminar as assumed in the derivation, Grashof numbers were calculated. Figure 2 shows the variation of N_{Gr} with x for parametric θ's. Eckert and Soehngen (*3*) found that for a vertical flat plate the incidence of turbulence for free-convection flow is characterized by a critical Grashof number of 4×10^8 (Figure 2).

For the first set of calculations, where θ is a constant and q''' a parameter, the values of the temperatures were taken as $T_w = 190$°F. and $T_e = 210$°F. Substituting $\theta = 20$°F into Equation (25) and using parametric values of q''', one obtained the curves of h/h_0 vs. x shown in Figure 3. Turbulence will occur at $x_{cr} = 3.2$, beyond which the calculations are not valid. Since maximum was obtained to the left of x_{cr} for $q''' = 200$ and $q''' = 300$, extrapolated expected results are shown as dotted lines for these two cases.

The physical explanation for the increase in heat transfer coefficient by the inclusion of a volume heat source with a constant θ or $T_e - T_w$ is shown in Figure 4, where curve 1 shows the temperature profile without a heat source. The inclusion of the heat source raises the temperature of the fluid at every point in the boundary layer, resulting in the higher temperature profile, curve 2. Since the heat transfer coefficients are being compared at constant $T_e - T_w$, δ_1 will be less than δ_0, and therefore $h/h_0 > 1$.

Figure 5 shows that for constant q''' an increase in θ lowers the curve of h/h_0 vs. x. This is explained in Figure 6, where curves 1 and 2 and the values δ_0, δ_1, T_W, and T_e are the same as in Figure 4. If the wall temperature is now lowered to T_w', curve 3 is the nonheat-source temperature profile. Including the heat source in the boundary layer will raise the temperature profile to curve 4. Since the value of the heat source remains constant at constant y, the differences between curves 1 and 2 and curves 3 and 4 are the same. The temperature increase between the latter curves is however of smaller relative magnitude to θ than in the case of the former curves. Thus curve 4 has the value T_e at a larger value of y than does curve 2; $\delta_0/\delta_1 > \delta_0/\delta_2$, and h/h_0 is less for the greater θ.

APPLICATION TO CYLINDER

The foregoing analysis for a vertical plate may be applied to a cylinder by the use of the procedure followed by Hermann (*4*), who established a hydrodynamic and thermal comparison between the two cases for free convection.

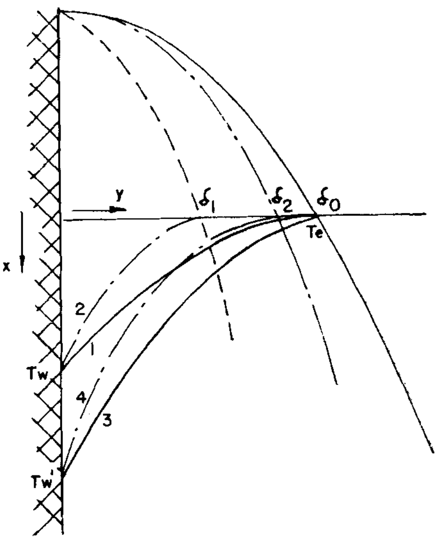

Fig. 6. Effect on the boundary-layer thickness of increasing the temperature difference at a constant heat-source value.

Although differences exist in the behavior of the boundary layer for each case, the differential equations are of similar form, and relations can therefore be set up between the two cases. By the introduction of calculated azimuth functions, Hermann represented cylinder variables in a dimensionless basis similar to those used for plates.

For equivalent hydrodynamic conditions, which he showed to exist when the Grashof number of the plate was equal to that of the cylinder, Hermann developed the following Nusselt number ratio:

$$\frac{N_{Nu}\,(\text{cylinder})}{N_{Nu}\,(\text{plate})} = 0.777$$

This is in fair agreement with an experimental ratio for air of 0.87 as given by Jakob (*5*). Since N_{Nu} is proportional to $N_{Gr}^{1/4}$, these ratios can be rearranged to yield the plate-height to cylinder-diameter ratio when the heat transfer coefficients are equal. Hermann's ratio yields a height to diameter ratio of 2.76, and Jakob's ratio yields 1.75. For this study a ratio of 2.5 was used as suggested by Eckert (*2*), which represents a compromise between experimental and analytical results.

In the application of the results it has been assumed that the ratio h/h_0 will remain unchanged for an equivalent cylinder system. For cylinders of practical dimensions the magnitude of the uncertainty in the plate-height to cylinder-diameter ratio has a minor effect on the results.

The average ratio of heat transfer coefficients from 0 to x on the plate, which is equivalent to the average from 0 to 2.5 D, is

$$\left(\frac{h}{h_0}\right)_{avg} = \frac{\int_0^{2.5D} \frac{h}{h_0}\, dx}{\int_0^{2.5D} dx} \qquad (28)$$

Thus for a ¼-in. O.D. horizontal cylinder with $\theta = 20°$F. and $q''' = 60$ B.t.u./(sec.)(cu. ft.), by graphical integration of Figure 3

$$\left(\frac{h}{h_0}\right)_{avg} = \frac{\int_0^{0.0521} \left(\frac{h}{h_0}\right) dx}{\int_0^{0.0521} dx} = 1.022$$

The limitation due to the incidence of turbulence will then determine the size of the cylinder that the results can be applied to for any θ.

For example for $\theta = 20°$F. the largest cylinder to which the results will apply is 1.28 in. in diameter; for $\theta = 100°$F. the largest cylinder is 0.720 in. in diameter.

LIMITATIONS ON RESULTS

The denominator of Equation (25) is an infinite series defined by Equation (8) which was truncated to permit evaluation. A general expression for $\Delta_n(X, Pr)$ must be determined from the solution of simultaneous equations of like coefficient arising from Equations (9) and (10). Direct analytical solutions for these equations however could not be obtained. Thus a general analytic term for $\Delta_n(X, Pr)$ and therefore for the series could not be obtained. This eliminated the application of the usual tests for convergence of infinite series.

However it was agreed (*6*) that the series was similar to one analyzed by Lefshetz and that it would indeed converge. Until the general term is found, though, perhaps numerically, the radius of convergence is indeterminate.

It should be recognized that the truncation used is quite common for an integral solution following the Karmann-Pohlhausen approach. Sparrow and Gregg (*9*) for example discuss convergence of a similar series and conclude that the truncation error is negligible.

CONCLUSIONS

1. With a volume heat source present in a fluid being cooled by free convection the heat transfer coefficient will be slightly higher than its value with no heat source present. This is due to the decrease in boundary-layer thickness caused by the inclusion of the volume heat source.
2. For parametric values of the temperature difference across the boundary layer an increase in the value of the volume heat source increases the ratio of the heat transfer coefficients (h/h_0) for a given cylinder diameter.
3. For parametric volume heat-source magnitudes an increase in the temperature difference across the boundary layer decreases the ratio of the heat transfer coefficients for a given cylinder diameter.
4. As the cylinder diameter increases, the incidence of turbulence occurs at smaller values of the temperature difference across the boundary layer.

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NOTATION

C = constant; C_1, C_2, etc., constant in the solution of differential equations.
C_p = heat capacity at constant pressure, B.t.u/(lb.)(°F.)
D = diameter, ft.
f_n = algebraic expression
g = acceleration due to gravity, ft./sec.²
g_n = algebraic expression
N_{Gr} = Grashof modulus (= $g\beta\theta x^3/\nu^2$)
h = heat transfer coefficient with volume heat source; h_0, without volume heat source, B.t.u./(sec.)(sq. ft./°F.)
k = thermal conductivity, B.t.u./(sec.)(sq. ft./°F./ft.)
K = heat-source function = $\left[\frac{\nu^{1/3}}{(g\beta)^{2/3}\theta^{5/3}}\right]\left[\frac{q'''}{\rho C_p}\right]$ dimensionless
N_{Pr} = Prandtl modulus (= ν/α), dimensionless
q = heat flux at a wall, B.t.u./(sq. ft.)(sec.)
q''' = volume heat source, B.t.u./(cu. ft.)(sec.)
r = distance in the radial direction, ft.
r_0 = outer radius of the cylinder, ft.
T = temperature at any point; T_w, wall temperature; T_1, reference temperature; T_e, temperature of the environment, °F.
u = velocity in the x direction, ft./sec.
x = distance; x_{cr}, critical distance for turbulent flow, ft.
X = distance function [= $(g\beta\theta/\nu^2)^{1/3}x$], dimensionless
y = distance, ft.

Greek Letters

α = Thermal diffusivity (= $k/\rho C_p$), sq. ft./sec.
β = density coefficient of expansion $\rho(t) = \rho(t_0)(1 - \beta(t - t_0))$, 1/°F.
δ = boundary-layer thickness; δ_0, without volume heat source; δ_1, δ_2, with volume heat source, ft.
Δ = boundary-layer thickness function [= $(g\beta\theta/\nu^2)^{1/3}$], dimensionless
$\Delta_0, \Delta_1, \cdots \Delta_n$ = coefficients of an infinite series
θ = Temperature function $(T_e - T_w)$; in body force term $(T - T_1)$, °F.
ν = kinematic viscosity, sq.ft./sec.
ρ = density, lb./cu. ft.
ϕ = temperature function $(T_e - T)$, °F.; angular coordinate in polar coordinate, radians.
ω = proportionality constant, ft./sec.
Ω = proportionality constant function = $[(g\beta\theta\nu)^{1/3}]$, dimensionless
$\Omega_0, \Omega_1, \cdots \Omega_n$ = coefficients of an infinite series

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